DO NOW

$\sum_{k=1}^{n} \frac{1}{n}(k^2+1) =$

$\frac{1}{n}(1^2+1)+\frac{1}{n}(2^2+1)+\frac{1}{n}(3^2+1)+\ldots+\frac{1}{n}(n^2+1)$

5.2 <u>Area</u>

Sigma Notation:

$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$

n ← upper bound

$i=1$ ← lower bound (doesn't have to be 1)

index of summation
↳ counting mechanism
(can be other letters)

Example: 1. $\sum_{k=3}^{7} k^2 = 3^2+4^2+5^2+6^2+7^2$

$9+16+25+36+49$

135

2. $\sum_{i=1}^{4} 5 = 5+5+5+5 = 20$

* value of 5 for each

3. $\sum_{i=1}^{5} 2i = 2(1)+2(2)+2(3)+2(4)+2(5) = 30$

$2(1+2+3+4+5)$

Properties of Summation: ★ Lower Bound must be 1

$\sum_{i=1}^{n} c = cn = nc$ (refer to #2)

$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$

$\sum_{i=1}^{n}(a_i \pm b_i)\ \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

Summation Formulas: ★ Lower Bound must be 1.

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Calculate: $\sum_{i=1}^{10} i = \frac{n(n+1)}{2} = \frac{10(11)}{2} = 55$

$\sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{\cancel{10}^{5}(11)(\cancel{21}^{7})}{\cancel{6}} = 385$

$\sum_{i=1}^{10} i^3 = \frac{n^2(n+1)^2}{4} = \frac{10^2 \cdot 11^2}{4} = \frac{100 \cdot 121}{4} = 3025$

*** Clear out all "clutter" around i, i^2, i^3

Example:

4. $\sum_{i=1}^{n} \frac{i^2}{n^3} = \frac{1}{n^3}\sum_{i=1}^{n} i^2$

$\frac{1}{\cancel{n^3}_{n^2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)$

$\frac{2n^2+3n+1}{6n^2}$

Now find $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3}$ $\lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2}$

$\lim_{n\to\infty} \frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}$

$\boxed{\frac{1}{3}}$

5. $\sum_{i=1}^{n} \frac{4i^2(i-1)}{n^4} = \frac{1}{n^4}\sum_{i=1}^{n} 4i^2(i-1)$

$\frac{1}{n^4}\sum_{i=1}^{n} 4i^3 - 4i^2$

$\frac{4}{n^4}\left[\sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} i^2\right]$

$\frac{4}{n^4}\left(\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}\right)$

$\frac{4}{n^4}\left(\frac{3n^2(n^2+2n+1) - 2n(2n^2+3n+1)}{12}\right)$

$\frac{4}{n^4}\left(\frac{3n^4+6n^3+3n^2-4n^3-6n^2-2n}{12}\right)$

$\frac{4}{n^4}\left(\frac{3n^4+2n^3-3n^2-2n}{12}\right)$

$\frac{\cancel{4}}{\cancel{n^4}_{n^3}}\left(\frac{\cancel{n}(3n^3+2n^2-3n-2)}{\cancel{12}_{3}}\right)$

$\boxed{\frac{3n^3+2n^2-3n-2}{3n^3}}$

6. $\sum_{j=1}^{n}(j+2)(j-5) = \sum_{j=1}^{n}(j^2-3j-10)$

$$\sum_{j=1}^{n} j^2 - 3\sum_{j=1}^{n} j - \sum_{j=1}^{n} 10$$

$$\frac{n(n+1)(2n+1)}{6} - 3\left(\frac{n(n+1)}{2}\right) - 10n$$

$$\frac{(n^2+n)(2n+1)}{6} - \frac{9(n^2+n)}{6} - \frac{60n}{6}$$

$$\frac{2n^3+3n^2+n-9n^2-9n-60n}{6}$$

$$\frac{2n^3-6n^2-68n}{6}$$

$$\frac{2n(n^2-3n-34)}{6}$$

$$\frac{n(n^2-3n-34)}{3}$$

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7. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right) = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{i=1}^{n}\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)$

$$\lim_{n\to\infty}\left(\frac{2}{n}\right)\left[\sum_{i=1}^{n} 1 + \frac{4}{n}\sum_{i=1}^{n} i + \frac{4}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$\lim_{n\to\infty}\left(\frac{2}{n}\right)\left[n + \frac{\overset{2}{\cancel{4}}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + \frac{\overset{2}{\cancel{4}}}{\underset{n}{\cancel{n^2}}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}\right)\right]$$

$$\lim_{n\to\infty}\left(\frac{2}{n}\right)\left[n+2n+2+\frac{2(2n^2+3n+1)}{3n}\right]$$

$$\lim_{n\to\infty}\left(\frac{2}{n}\right)\left[\frac{3n^2+6n^2+6n+4n^2+6n+2}{3n}\right]$$

$$\lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{13n^2+12n+2}{3n}\right)$$

$$\lim_{n\to\infty}\left(\frac{26n^2+24n+4}{3n^2}\right)$$

$$\boxed{\frac{26}{3}}$$

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HOMEWORK

pg 303 - 304; 1 - 11 odd, 15 - 20,
31 - 34, 35 - 43 odd

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